

Creation of Magnetic Fields by Electrostatic and Thermal Fluctuations

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(Dated: September 01, 2009)

It is pointed out that the unmagnetized inhomogeneous plasmas can support a low frequency electromagnetic ion wave as a normal mode like Alfvén wave of magnetized plasmas. But this is a coupled mode produced by the mixing of longitudinal and transverse components of perturbed electric field due to density inhomogeneity. The ion acoustic wave does not remain electrostatic in non-uniform plasmas. On the other hand, a low frequency electrostatic wave can also exist in the pure electron plasmas. But the magnetic field fluctuations in both electron as well as in electron-ion plasmas are coupled with the electrostatic perturbations in unmagnetized case. The main instability condition for these low frequency electrostatic and electromagnetic modes is the same $\frac{2}{3}\kappa_n < \kappa_T$ (where κ_n and κ_T are inverse of the scale lengths of density and electron temperature, respectively).

PACS numbers:

I. INTRODUCTION

A low frequency electromagnetic wave has been investigated as a fundamental normal mode of unmagnetized inhomogeneous plasmas. The ion acoustic wave (IAW) is a well-known low frequency mode of unmagnetized plasmas but it is purely electrostatic. Even for lightest ions of hydrogen the electron to ion mass ratio is very small $\frac{m_e}{m_i} \simeq 10^{-3} \ll 1$. In the limit $m_e \rightarrow 0$, the electrons are assumed to follow the Boltzmann density distribution in the electrostatic field of the IAW.

In fact the electron inertia can play a vital role in producing a low frequency electromagnetic

wave. More than a decade ago [1], it was proposed that an electromagnetic wave having frequency near IAW is a normal mode of unmagnetized plasmas which can be responsible for magnetic field generation if it becomes unstable. In the derivation of its linear dispersion relation the electron inertia was not ignored while the displacement current was neglected. It was shown that the compressibility and vorticity can couple due to density inhomogeneity and hence a low frequency electromagnetic wave can be produced. Basically electrostatic IAW and magnetostatic mode [2] cooperate with each other to develop such a wave. The electron temperature perturbation was not taken into account and the steady state was assumed to be maintained by external mechanisms. The theory was applied to explain the magnetic field generation in laser plasmas. But the longitudinal and transverse characters of electric field decouple if the quasi-neutrality is used [1]. Then both the high frequency and low frequency electromagnetic instabilities were also investigated in unmagnetized plasmas [3].

The interest in the investigation of low frequency magnetic fluctuations in unmagnetized plasmas was initiated after the first experimental observation of magnetic field generated in a laser induced plasma [4]. Then more experiments were performed [5, 6] on these lines. Several theoretical models were presented to explain the magnetic field generation in laser produced plasmas [7–13].

The magnetic electron drift vortex (MEDV) mode was proposed as a pure transverse linear mode which can exist because of the electron temperature fluctuations in unmagnetized inhomogeneous electron plasmas [13]. This mode can become unstable [14] if the equilibrium electron temperature gradient is parallel to the density gradient and it is maintained by external effects. The basic MEDV mode is believed to exist in an electron plasma with smooth gradients. However, it has also been shown that the instability of MEDV mode can arise when the density profile is represented by a single step connecting two regions of nonzero density and temperature gradients [15]. The instabilities of magnetic and acoustic waves driven by perturbed baroclinic vector have also been investigated in a pure electron plasma [16]. Recently the nonlinear evolution of two dimensional MEDV modes has been studied using computer simulation [17]. The spontaneous magnetic field generation and formation of nonlinear structures has been discussed in detail. Most of the mechanisms proposed to explain magnetic fluctuations in initially unmagnetized plasmas are based on electron mag-

netohydrodynamics (EMHD) which has been discussed in detail in Refs. [18, 19]. Some weaknesses and contradictions of EMHD model were pointed out several years ago [20]. The long-lived and slowly propagating nonlinear whistler structures (NLWS) or whistler spheromaks (WSPS) have also been studied [21] using EMHD equations. Such structures have been observed in magnetized laboratory plasmas [22, 23].

In MEDV mode the divergence of electric field is assumed to be zero ($\nabla \cdot \mathbf{E}_1 = 0$) while the divergence of electron velocity is non-zero ($\nabla \cdot \mathbf{v}_{e1} \neq 0$). Furthermore the ions are treated to be stationary. The frequency ω of the MEDV mode is assumed to lie in between the ion plasma frequency and electron plasma frequency i.e. $\omega_{pi} \ll \omega \ll \omega_{pe}$ where $\omega_{pj} = \left(\frac{4\pi n_{0j} e^2}{m_j} \right)$, for $j=e,i$ and c is speed of light while k is the wave vector. These restrictions and assumptions are indeed very strict [20] and are not fulfilled in general.

Several authors have considered the role of ion dynamics in the magnetic instabilities in unmagnetized plasmas. The effects of ion dynamics on MEDV mode have been investigated in the frame work of local approximation [24]. The coupling of high frequency electromagnetic wave with low frequency ion acoustic wave has been discussed in certain limits [3]. The coupling of magnetic fluctuations with ion acoustic wave has been discussed in a plasma with the steady state given as $\nabla p_{e0} = 0$ [25] without considering electron temperature perturbation. But the group velocity turns out to be negative in this treatment.

It is important to find out some electromagnetic mode taking into account the ion dynamics using minimum approximations so that the strict restrictions on the frequency and wavelength of the perturbation are relaxed. It is better if the only required condition on frequency becomes $\omega \ll \omega_{pe}, ck$. If we assume a steady state as $\nabla p_{e0} = 0$, then the temperature gradient becomes anti-parallel to density. But laser and astrophysical plasmas are open systems and many external mechanisms can maintain a study state with parallel density and temperature gradients. For example, in stellar cores, both the density and temperature increase towards centre of the star due to fusion and star is held intact because of gravity. Therefore both the cases of parallel and anti-parallel gradients should be discussed. The electron thermal fluctuations can produce electromagnetic wave as was proposed many decades ago [13] but the assumptions used in this work are very restrictive.

It is not necessary to assume a pure transverse perturbation in an electron plasma. Rather the perturbation can be partially transverse and partially longitudinal and hence the electron

density perturbation may not be neglected. The frequency of such a wave in electron plasma turns out to be near $(\lambda_e k_y) v_{te} \kappa_n$ where $v_{te} = \left(\frac{T_e}{m_e}\right)^{\frac{1}{2}}$ is the electron thermal speed, $\lambda_e = \frac{c}{\omega_{pe}}$ is the electron skin depth and $\kappa_n = \left|\frac{1}{n_0} \frac{dn_0}{dx}\right|$ is the inverse of density gradient scale length $L_n = \frac{1}{\kappa_n}$. In the local approximation we need to have $\kappa_n \ll k$. Furthermore the condition $\omega_{pi}^2 \ll v_{te}^2 k^2$ can be satisfied if $\frac{m_e}{m_i} \ll \lambda_{De}^2 k_y^2$. Therefore, generally we may have $v_{te} \kappa_n \lesssim \omega_{pi}$ and hence one cannot neglect ion dynamics. Moreover, $v_{te} \kappa_n \simeq c_s k$ (where $c_s = \left(\frac{T_e}{m_i}\right)^{\frac{1}{2}}$ is ion sound speed) is also possible. Therefore, it is expected that the electron thermal fluctuations can couple with IAW to produce stable and unstable low frequency electromagnetic waves in unmagnetized plasmas. It will be shown that the dispersion relation of ion acoustic wave is modified in the inhomogeneous plasma due to the coupling of electrostatic and magnetic fluctuations. A low frequency electrostatic mode can also exist in a non-uniform pure electron plasma. This mode can couple with the IAW in electron-ion plasma. Several low frequency electrostatic and electromagnetic waves of inhomogeneous unmagnetized electron and electron-ion plasmas are investigated. Interestingly the main instability condition for these modes is the same.

II. LOW FREQUENCY ELECTROMAGNETIC WAVES IN ELECTRON PLASMAS

Let us consider the electron plasma in the background of stationary ions. First we discuss the dispersion relation of pure transverse MEDV mode [13, 14]. Then we show that the compressibility cannot be neglected. Finally linear dispersion relations of low frequency electrostatic and electromagnetic perturbations are obtained in an electron plasma. The set of equations for MEDV mode in the linear limit can be written as,

$$m_e n_0 \partial_t \mathbf{v}_{e1} = -en_0 \mathbf{E}_1 - \nabla p_{e1} \quad (1)$$

$$\nabla \times \mathbf{B}_1 = \frac{4\pi}{c} \mathbf{J}_1 \quad (2)$$

$$\mathbf{J}_1 = -en_0 \mathbf{v}_{e1} \quad (3)$$

$$\nabla \times \mathbf{E}_1 = -\frac{1}{c} \partial_t \mathbf{B}_1 \quad (4)$$

Since $p_1 = n_0 T_{e1}$ therefore energy equation becomes,

$$\frac{3}{2} n_0 \partial_t T_{e1} + \frac{3}{2} n_0 (\mathbf{v}_{e1} \cdot \nabla) T_{e0} = -p_0 \nabla \cdot \mathbf{v}_{e1} \quad (5)$$

Curl of (1) gives,

$$\partial_t(\nabla \times \mathbf{v}_{e1}) = \frac{e}{m_e c} \partial_t \mathbf{B}_1 + \frac{1}{m_e n_0} (\nabla n_0) \times \nabla T_{e1} \quad (6)$$

Equations (2) and (3) yield,

$$\mathbf{v}_{e1} = -\frac{c}{4\pi e n_0} \nabla \times \mathbf{B}_1 \quad (7)$$

and hence

$$\nabla \times \mathbf{v}_{e1} = \frac{c}{4\pi e n_0} \nabla^2 \mathbf{B}_1 \quad (8)$$

where $\nabla n_0 \times (\nabla \times \mathbf{B}_1) = 0$ due to the assumption $\mathbf{k} \perp \nabla n_0 \perp \mathbf{B}_1$. Equation (8) predicts $\mathbf{E}_1 = E_1 \hat{\mathbf{x}}$ while $\nabla n_0 = \hat{\mathbf{x}} \frac{dn_0}{dx}$, $\nabla = (0, ik_y, 0)$ and $\mathbf{B}_1 = B_1 \hat{\mathbf{z}}$ have been chosen.

Equations (6) and (8) yield,

$$(1 + \lambda_e^2 k^2) \partial_t \mathbf{B}_1 = -\frac{c}{en_0} (\nabla n_0 \times \nabla T_{e1}) \quad (9)$$

where $\lambda_e = \frac{c}{\omega_{pe}}$. Equation (7) yields,

$$\nabla \cdot \mathbf{v}_{e1} = \frac{c}{4\pi n_0 e} \frac{\nabla n_0}{n_0} \cdot (\nabla \times \mathbf{B}_1) \quad (10)$$

and therefore one obtains,

$$T_{e1} = \frac{2}{3} \frac{c}{4\pi n_0 e} k_y \kappa_n B_1 \quad (11)$$

Then (9) and (11) give the linear dispersion relation of MEDV mode as,

$$\omega^2 = \frac{2}{3} C_0 \left(\frac{\kappa_n}{k_y} \right)^2 v_{Te}^2 k_y^2 \quad (12)$$

where $C_0 = \frac{\lambda_e^2 k^2}{1 + \lambda_e^2 k^2}$ and $v_{te} = (T_e/m_e)^{\frac{1}{2}}$. The geometry of MEDV mode in cartesian coordinates is shown in Fig. 1.

If $\nabla T_{e0} \neq 0$ is assumed, then (12) becomes,

$$\omega^2 = C_0 \frac{\kappa_n}{k_y} \left[\frac{(\frac{2}{3} \kappa_n - \kappa_T)}{k_y} \right] v_{Te}^2 k^2 \quad (13)$$

where $\kappa_T = |\frac{1}{T_{e0}} \frac{dT_{e0}}{dx}|$ and $\nabla T_{e0} = +\mathbf{x} \frac{dT_{e0}}{dx}$ has been used. If $(\frac{2}{3} \kappa_n < \kappa_T)$, then the mode becomes unstable [14].

It can be noticed that $\nabla \cdot \mathbf{E}_1 = 0$ and $\nabla \cdot \mathbf{v}_{e1} \neq 0$ have been assumed in the above treatment and it does not seem to be very convincing. If we write (1) in x and y components, we can

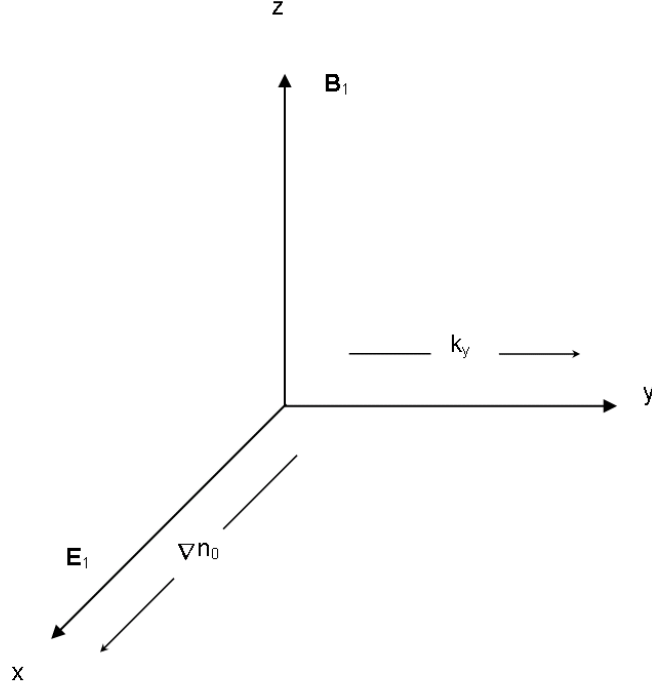


FIG. 1: The MEDV mode geometry shows that it is a pure transverse mode while the electron equation of motion indicates $E_{y1} \neq 0$ and hence $\mathbf{k} \cdot \mathbf{E}_1 \neq 0$.

observe that the y-component of \mathbf{E}_1 should not be considered as zero due to $\partial_y p_{e1} \neq 0$ and hence the wave may be partially transverse and partially longitudinal. Equation (1) yields,

$$\partial_t \nu_{ex1} = -\frac{e}{m_e} E_{x1} - v_{Te}^2 \left\{ \kappa_n \frac{T_{e1}}{T_0} + \kappa_T \frac{n_{e1}}{n_0} \right\} \quad (14)$$

and

$$\partial_t \nu_{ey1} = -\frac{e}{m_e} E_{y1} - v_{Te}^2 \left\{ i k_y \left(\frac{T_{e1}}{T_0} + \frac{n_{e1}}{n_0} \right) \right\} \quad (15)$$

It is obvious from (15), that $\nabla \cdot \mathbf{v}_{e1} \neq 0$ which implies $E_{y1} = -\partial_y \varphi_1 \neq 0$. It is clear from Fig. 1 that MEDV mode should be revisited including longitudinal effects.

The curl of (2) yields a relation between E_{1x} and E_{1y} for $\omega \ll \omega_{pe}$ as,

$$E_{1x} = -\frac{1}{a} \frac{\kappa_n}{k_y} (i E_{1y}) \quad (16)$$

where $a = (1 + \lambda_e^2 k_y^2)$.

Using equation of motion (1) instead of (10) in equation (5), we find,

$$W_0^2 \frac{T_{e1}}{T_0} = v_{te}^2 k_y^2 \left(1 - \frac{3}{2} \frac{\kappa_T}{k_y^2} - \Gamma_0^2 \right) \frac{n_{e1}}{n_0} - \frac{e}{m_e} \left(i k_y E_{1y} + \frac{3}{2} \kappa_T E_{1x} \right) \quad (17)$$

where $W_0^2 = \frac{3}{2} \omega^2 - v_{te}^2 k_y^2 \left(1 - \frac{3}{2} \frac{\kappa_T \kappa_n}{k_y^2} \right)$ and $\Gamma_0^2 = \frac{(\kappa_T - \kappa_n) \kappa_T}{k_y^2}$.

The continuity equation yields,

$$\begin{aligned} L_0^2 \frac{n_{e1}}{n_0} = & - \left(1 + \frac{v_{te}^2 k_y^2}{W_0^2} (1 - \kappa_n^2 / k_y^2) \right) \left(i \frac{e}{m_e} k_y E_{1y} \right) \\ & - \left\{ 1 + \frac{3}{2} \frac{\kappa_T}{\kappa_n} \frac{v_{te}^2 k_y^2}{W_0^2} (1 - \kappa_n^2 / k_y^2) \right\} \left(\frac{e}{m_e} \kappa_n E_{1x} \right) \end{aligned} \quad (18)$$

where $L_0^2 = \left\{ \omega^2 - v_{te}^2 k_y^2 (1 - \kappa_n^2 / k_y^2) - \frac{v_{te}^4 k_y^4}{W_0^2} (1 - \kappa_n^2 / k_y^2) \left(1 - \frac{3}{2} \frac{\kappa_T}{k_y^2} - \Gamma_0^2 \right) \right\}$. The Poisson equation

$$\nabla \cdot \mathbf{E}_1 = -4\pi e \left(\frac{n_{e1}}{n_0} \right) \quad (19)$$

can be written as,

$$\begin{aligned} i k_y E_{1y} [L_0^2 W_0^2 - \omega_{pe}^2 \{W_0^2 + v_{te}^2 k_y^2 (1 - \kappa_n^2 / k_y^2)\}] \\ = (\kappa_n E_{1x}) \omega_{pe}^2 \left[W_0^2 + \frac{\kappa_T}{\kappa_n} v_{te}^2 k_y^2 (1 - \kappa_n^2 / k_y^2) \right] \end{aligned} \quad (20)$$

Equations (16) and (20) yield a linear dispersion relation in the limit $\omega^2 \ll \omega_{pe}^2$ as,

$$\begin{aligned} a [L_0^2 W_0^2 - \omega_{pe}^2 \{W_0^2 + v_{te}^2 k_y^2 (1 - \kappa_n^2 / k_y^2)\}] \\ = -\omega_{pe}^2 (\kappa_n / k_y)^2 \left[W_0^2 + \frac{3}{2} \frac{\kappa_T}{\kappa_n} v_{te}^2 k_y^2 (1 - \kappa_n^2 / k_y^2) \right] \end{aligned} \quad (21)$$

This equation can be simplified as,

$$H_0 W^2 = -\frac{2}{3} v_{te}^2 \kappa_n^2 \left[\left(1 - \frac{3}{2} \frac{\kappa_T}{\kappa_n} \right) - (1 + \lambda_e^2 k_y^2) \left(1 - \frac{3}{2} \frac{\kappa_T}{\kappa_n} \right) \right] \quad (22)$$

where

$$H_0 = \left[\left\{ (1 + \lambda_{De}^2 k_y^2) + \frac{2}{3} \lambda_{De}^2 k_y^2 \right\} a - \kappa_n^2 / k_y^2 \right]$$

The second term in right hand side of (22) will disappear if $E_{1y} = 0$ is assumed. The first term and the factor $\lambda_e^2 k_y^2$ in second term are the contributions of transverse components

E_{1x} . Thus one can not obtain MEDV-mode dispersion relation for $E_{1y} = 0$ from (22). It is interesting to note that equation (22) can be simplified to obtain,

$$\omega^2 = \frac{2}{3H_0} \lambda_e^2 k_y^2 (v_{te}^2 \kappa_n) \left(1 - \frac{3}{2} \kappa_T / \kappa_n \right) \quad (23)$$

which looks very similar to MEDV mode dispersion relation.

However, the instability condition for this electromagnetic mode is the same as was for MEDV mode that is

$$\frac{2}{3} \kappa_n < \kappa_T \quad (24)$$

The low frequency mode (23) is partially transverse and partially longitudinal.

If $E_{1x} = 0$ is assumed, then equation (21) yields a low frequency electrostatic wave in a non-uniform unmagnetized plasma with the dispersion relation,

$$\omega^2 = \frac{v_{te}^2 \kappa_n^2 \left(\frac{2}{3} - \kappa_T / \kappa_n \right)}{\left[(1 + \lambda_{De}^2 k_y^2) + \frac{2}{3} \lambda_{De}^2 k_y^2 \right]} \quad (25)$$

The instability condition remains the same (24). This indicates that the instability predicted by the so called pure transverse MEDV mode is always coupled with electrostatic perturbations in electron plasmas.

In electron-ion plasmas, these modes of equations (23) and (25) can couple with the ion acoustic wave.

III. IAW AND MAGNETOSTATIC MODE

Here we shall show that transverse magnetostatic mode [2] which is obtained in the limit $\omega^2 \ll \omega_{pe}^2$ can couple with IAW in a nonuniform plasma [1]. Therefore, both ions and electrons are considered to be dynamic while the electron temperature perturbation is neglected. The ions are assumed to be cold for simplicity and therefore equation of motion becomes,

$$\partial_t \mathbf{v}_{i1} = \frac{e}{m_i} \mathbf{E}_1 \quad (26)$$

The continuity equation yields,

$$\frac{n_{i1}}{n_0} = \frac{e}{m_i \omega^2} (\kappa_n E_{1x} + i k_y E_{1y}) \quad (27)$$

If quasi-neutrality is used then the transverse component E_{1x} and longitudinal component E_{1y} become uncoupled [1]. Therefore, we assume $\lambda_{De}^2 k_y^2 \neq 0$ and use Poisson equation which in the limit $\omega^2 \ll \omega_{pe}^2$ becomes,

$$[-v_{te}^2 k_y^2 \omega^2 - \omega_{pi}^2 (\omega^2 - v_{te}^2 k_y^2) - \omega_{pe}^2 \omega^2] i k_y E_{1y} \simeq [\omega_{pi}^2 (\omega^2 - v_{te}^2 k_y^2) + \omega_{pe}^2 \omega^2] \kappa_n E_{1x} \quad (28)$$

Note that the term $v_{te}^2 k_y^2$ is not ignored compared to ω_{pe}^2 to couple E_{1x} and E_{1y} .

Equations (16) and (28) give a linear dispersion relation as,

$$\omega^2 = \frac{c_s^2 k_y^2 (a - \frac{\kappa_n^2}{k_y^2})}{(ab - \frac{\kappa_n^2}{k_y^2})} \quad (29)$$

where $c_s^2 = \frac{T_e}{m_i}$, and $b = (1 + \lambda_{De}^2 k_y^2)$. We have to use Poisson equation to obtain a quadratic equation in ω while (2) implies $n_{e1} \simeq n_{i1}$. The equation (29) is the same as equation (21) of Ref. [1] where two small terms in the denominator are missing. Note that $\lambda_{De}^2 < \lambda_e^2$ and if quasi neutrality is used due to Ampere's law, then (29) yields the basic electrostatic IAW dispersion relation $\omega^2 = c_s^2 k_y^2$. If the displacement current is retained and Poisson equation is used without using $\omega^2, \omega_{pi}^2 \ll \omega_{pe}^2$, then one obtains a dispersion relation of coupled three waves; ion acoustic wave, electron plasma wave and high frequency transverse wave [3].

Actually the contribution of displacement current in the curl of Maxwell's equation has been neglected for $\omega^2 \ll \omega_{pe}^2, c^2 k^2$. This should not mean that the electrostatic part of current is also divergence free, in our opinion. In the divergence part of Maxwell's equation $\omega^2 \ll \omega_{pe}^2$ is used but $v_{te}^2 k_y^2$ term is assumed to be important. It may be mentioned that in this treatment, Ampere's law does not imply quasi-neutrality necessarily. We need a coupling of divergence part and curl part of the current and for this we need to assume $\frac{m_e}{m_i} < \lambda_{De}^2 k_y^2$ in the limit $\omega^2 \ll \omega_{pe}^2, c^2 k^2$.

IV. LOW FREQUENCY ELECTROMAGNETIC ION WAVES

Now we present a simple but interesting theoretical model for low frequency electromagnetic waves assuming ions to be cold. The electrostatic waves will also be considered and it will be shown that magnetic field perturbation is coupled with the dominant electrostatic field. In the low frequency limit $|\partial_t| \ll \omega_{pe}, ck$, the electrons are commonly assumed to be inertial-less, i.e. $\frac{m_e}{m_i} \rightarrow 0$. Then the longitudinal and transverse components of electric field

decouple. For IAW it is assumed that $E = -\nabla\varphi$ while electron equation of motion yields Boltzmann density distribution as

$$\frac{n_e}{n_0} \simeq e^{-\frac{e\varphi}{T_e}} \quad (30)$$

Then the fundamental low frequency mode of the plasma turns out to be the ion acoustic wave with linear dispersion relation

$$\omega_s^2 = c_s^2 k_y^2 \quad (31)$$

in the quasi-neutrality limit. If dispersion effects are included, then instead of (31) one obtains,

$$\omega_s^2 = \frac{c_s^2 k_y^2}{1 + \lambda_{De}^2 k_y^2} \quad (32)$$

In the limit $1 \ll \lambda_{De}^2 k_y^2$, equation (32) gives ion plasma oscillations $\omega^2 = \omega_{pi}^2$. It is important to note that in the presence of inhomogeneity, a new scale $\frac{\kappa_n}{k_y}$ is added to the system. If $\frac{m_e}{m_i} \ll \left(\frac{\kappa_n}{k_y}\right)^2$, then longitudinal and transverse components of electric field can couple to generate low frequency electromagnetic waves. The divergence and curl of (1) give, respectively,

$$\partial_t \nabla \cdot (n_0 \mathbf{v}_{e1}) = -\frac{e}{m_e} n_0 \nabla \cdot \mathbf{E}_1 - \frac{e}{m_e} \nabla n_0 \cdot \mathbf{E}_1 - \frac{1}{m_e} (\nabla \cdot \nabla p_{e1}) \quad (33)$$

and

$$\partial_t (\nabla \times \mathbf{v}_{e1}) + (\kappa_n \times \partial_t \mathbf{v}_{e1}) = -\frac{e}{m_e} \kappa_n \times \mathbf{E}_1 - \frac{e}{m_e} \nabla \times \mathbf{E}_1 \quad (34)$$

where $\kappa_n = |\frac{1}{n_0} \frac{dn_0}{dx}|$ and $\nabla n_0 = +\hat{\mathbf{x}} |\frac{dn_0}{dx}|$ has been assumed. If initially electric field was purely electrostatic i.e. $\mathbf{E}_1 = -\nabla\varphi_1$, then it will develop a rotating part as well if $\nabla n_0 \times \mathbf{E}_1 \neq 0$, as is indicated by the right hand side (RHS) of (34).

The Poisson equation in this case is,

$$\nabla \cdot \mathbf{E}_1 = 4\pi n_0 e \left(\frac{n_{i1}}{n_0} - \frac{n_{e1}}{n_0} \right) \quad (35)$$

Using (18) and (27), the above equation can be written as,

$$\begin{aligned} ik_y E_{1y} & \left[L_0^2 W_0^2 \omega^2 - L_0^2 W_0^2 \omega_{pi}^2 - \omega_{pe}^2 W_0^2 + v_{te}^2 k_y^2 \left(1 - \frac{\kappa_n^2}{k_y^2} \right) \right] \\ & = \kappa_n E_{1x} \left[L_0^2 W_0^2 \omega_{pi}^2 + \omega_{pe}^2 \left\{ W_0^2 + \frac{3}{2} \frac{\kappa_T}{\kappa_n} v_{te}^2 k_y^2 \left(1 - \frac{\kappa_n^2}{k_y^2} \right) \right\} \right] \end{aligned} \quad (36)$$

Then (16) and (36) yield,

$$a \left[L_0^2 W_0^2 - \omega_{pe}^2 (W_0^2 + v_{te}^2 k_y^2) \left(1 - \kappa_n^2 / k_y^2 \right) \right]$$

$$\begin{aligned}
&= -\frac{\kappa_n^2}{k_y^2} \left[\omega_{pe}^2 \left\{ W_0^2 + \frac{3}{2} \frac{\kappa_T}{\kappa_n} v_{te}^2 (1 - \kappa_n^2/k_y^2) \right\} \right] \\
&\quad + \frac{L_0^2 W_0^2}{\omega^2} \omega_{pi}^2 \left(a - \frac{\kappa_n^2}{k_y^2} \right)
\end{aligned} \tag{37}$$

In the limit $\omega^2, \omega_{pi}^2 \ll \omega_{pe}^2$, (37) gives a linear dispersion relation,

$$\omega^2 = \frac{1}{H_0} \left[(\lambda_e^2 k_y^2) v_{te}^2 \kappa_n^2 \left(\frac{2}{3} - \frac{\kappa_T}{\kappa_n} \right) + (a - \kappa_n^2/k_y^2) c_s^2 k_y^2 \left\{ \frac{5}{3} - \left(\frac{\kappa_T^2}{k_y^2} + \frac{\kappa_T \kappa_n}{k_y^2} \right) \right\} \right] \tag{38}$$

If ion dynamics is ignored then (38) reduces to (23). The above equation shows a coupling of ion acoustic wave with the electromagnetic fluctuations in nonuniform unmagnetized plasmas.

In the electrostatic limit ($E_{1x} = 0$), equation (38) becomes,

$$\omega^2 = \frac{1}{H_1} \left[v_{te}^2 \kappa_n^2 \left(\frac{2}{3} - \kappa_T/\kappa_n \right) + c_s^2 k_y^2 \left\{ \frac{5}{3} + \left(\frac{\kappa_T^2}{k_y^2} + \frac{\kappa_T \kappa_n}{k_y^2} \right) \right\} \right] \tag{39}$$

where $H_1 = \{(1 + \lambda_{De}^2 k_y^2) + \frac{2}{3} \lambda_{De}^2 k_y^2\}$. For stationary ions, (39) reduces to (25).

It is important to note that the main instability conditions for electrostatic ion acoustic wave (39) and low frequency electromagnetic wave (38) is again the same (24).

In electron-ion plasma, the electromagnetic wave dispersion relation in the quasi-neutrality limit can be written as,

$$\omega^2 = \frac{\lambda_e^2 k_y^2}{(a - \kappa_n^2/k_y^2)} v_{te}^2 \kappa_n^2 \left(\frac{2}{3} - \frac{\kappa_T}{\kappa_n} \right) + \frac{5}{2} c_s^2 k_y^2 \tag{40}$$

The wave geometry is shown in Fig. 2.

The instability can occur if (24) is satisfied along with

$$\frac{5}{2} c_s^2 k_y^2 < \frac{\lambda_e^2 k_y^2}{(a - \kappa_n^2/k_y^2)} v_{te}^2 \kappa_n^2 \tag{41}$$

V. DISCUSSION

The theoretical model presented here shows that several electrostatic and electromagnetic low frequency waves can exist in un-magnetized electron as well as electron-ion plasmas. Interestingly the main instability condition for these modes is the same $\frac{2}{3} \kappa_n < \kappa_T$ where κ_n and

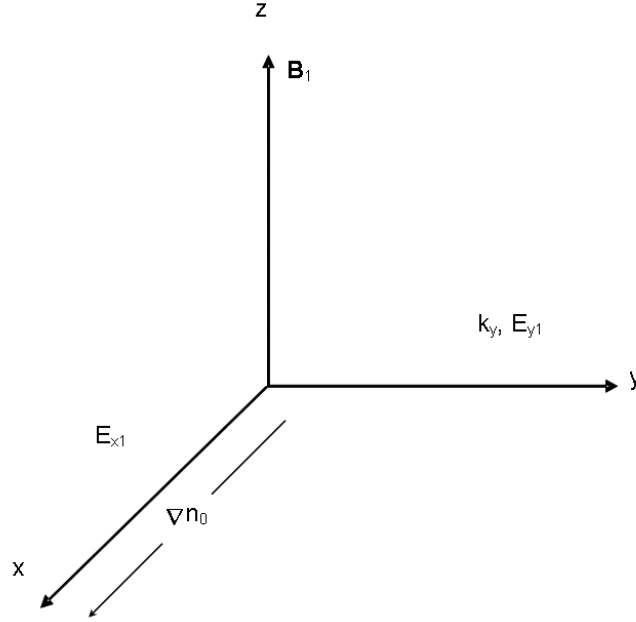


FIG. 2: The simplest possible geometry of electromagnetic ion wave is shown. This low frequency wave is partially longitudinal and partially transverse.

κ_T are the inverse of density and electron temperature gradient scale lengths, respectively. This indicates that the magnetic field fluctuations are always coupled with the electrostatic perturbations.

It is well-known that the ion acoustic wave is a fundamental low frequency electrostatic mode of un-magnetized plasmas. Here we have found that the inhomogeneous electron plasma can also support a low frequency electrostatic mode. This electrostatic mode can couple with the magnetic field perturbations to give rise to a partially electrostatic and partially transverse wave. Therefore instead of the so called magnetic electron drift vortex (MEDV) mode there exists a low frequency electromagnetic wave having both the contributions of longitudinal and transverse electric field components as has been shown in equations (22). The dispersion relation of MEDV mode is very similar to the electromagnetic wave discussed in

this investigation as can be seen in equation (23). But the important point to note is that this dispersion relation appears after a cancellation of two terms. One of these terms is a part of longitudinal electric field E_{1y} . Moreover, if transverse electric field component E_{1x} is neglected one obtains a pure electrostatic wave of equation (25). The frequency range of both the modes is very close to each other. The instability conditions are almost the same. This fact strengthens the view point that magnetic fluctuations are coupled with the dominant electrostatic fields.

Any initial electrostatic field perturbation can produce its transverse component in the presence of density gradient [1, 3]. This phenomenon can cause a coupling of ion acoustic wave (IAW) with the low frequency transverse magnetostatic mode. This coupled mode has already been investigated more than a decade ago [1]. But it can exist in a relatively shorter wavelength range for $\frac{m_e}{m_i} < \lambda_{De}^2 k^2$. In the quasi-neutrality limit, the IAW and magnetostatic modes decouple.

In case of the electron-ion plasmas, the IAW does not remain electrostatic in inhomogeneous plasmas [1, 25, 26]. The electromagnetic mode discussed for the case of pure electron plasma can couple with ion acoustic wave as shown in equation (40). Similar to the electrostatic IAW, this low frequency electromagnetic ion wave can exist even in the quasi-neutrality limit. The electrostatic and electromagnetic waves discussed here can become unstable if the density and temperature gradients are parallel to each other which can be the case in laser plasmas similar to stellar cores.

These low frequency waves can be the intrinsic source of magnetic fields in stars, galaxies as well as in laser plasmas. The main instability condition of the several electrostatic and electromagnetic waves discussed here is the same as given in the form of inequality (24). Therefore, in our opinion, electrostatic perturbations are strongly coupled with magnetic field fluctuations in inhomogeneous un-magnetized plasmas.

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